junction instead of the ideal one. At large direct magnetic fields it is possible to observe resonance loss for one of the split modes.

The split frequencies are also related to the loaded Q-factor by (1). This allows the loaded Q-factor to be determined.

VI. Susceptance Slope Parameter of Partially Magnetized Junction

One way in which the susceptance slope parameter of a partially magnetized junction may be obtained is by using the universal admittance equation of a junction given by (2).

The susceptance slope parameter is immediately obtained from this last equation by independently measuring the input admittance and the two split frequencies. Methods of measuring these last two quantities have already been described in the text.

It is also observed from this last equation that g is an increasing function of the magnetic field as long as the splitting of the resonant modes is widening.

VII. Experimental Results

This section gives experimental results obtained on a below-resonance stripline circulator using the techniques developed here. The schematic of the stripline circulator investigated is shown in Fig. 4. The junction used here consists of a garnet disk surrounded by a dielectric sleeve. The magnetization of the garnet material used was 0.0500 Wb/m^2 and its dielectric constant was $\epsilon_r = 14.7$. The dielectric constant of the ring was $\epsilon_r = 8.0$. The diameter of the garnet disk was 12.5 mm and the outside diameter of the ring was 25.0 mm. The thickness of the garnet disk was 2.54 mm. The experimental results obtained here are shown in Figs. 5–7. Fig. 5 shows the gyrator conductance of this junction as a function of the direct magnetic field using the method developed in Section III of this text. Fig. 6 shows the two split frequencies of this geometry obtained by using the technique derived in Section V. Finally, Fig. 7 gives the susceptance slope parameter of this junction as a function of the direct magnetic field. This last illustration is obtained by solving the universal gyrator equation for the susceptance slope parameter in terms of the experimental gyrator conductance and split frequencies of the magnetized junction. Fig. 7 also indicates that the susceptance slope parameter of the junction is independent of the direct magnetic field which is as it should be for the dielectric loaded junction used here.

VIII. Conclusions

This short paper has given new simple ways of measuring each of the three parameters which enter into the admittance equation of junction circulators. The methods described require no phase information and are therefore ideally suited for reflectometer-type measurements. All measurements described in this short paper are made in the input transmission line of the junction with the other two ports connected to similar transmission lines terminated in their characteristic impedance. The results obtained here apply to lossless circulators for which the two resonant modes are symmetrically split by the magnetic field, and for which the frequency variation of the third mode can be omitted.

Acknowledgment

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End Effects of Half-Wave Stripline Resonators

ROLF O. E. LAGERLÖF

***Abstract*—The end effects of an open-circuited TEM transmission line make the line electrically longer than its physical length. In this short paper a half-wave resonator of a balanced strip transmission line has been analyzed and the required foreshortening of the line to achieve a prescribed resonance frequency has been calculated. Also the decrease in the characteristic impedance of the stripline caused by the end effects has been determined. The theory is in reasonably good agreement with measurements performed, especially for narrow stripline resonators.**

By using an electrostatic theory Altschuler and Oliner [1] calculated the foreshortening for a strip of infinite width. When the physical length a of the strip exceeds twice the ground-plane spacing b, the foreshortening obtained this way is

$$\Delta a \approx 0.44b. \tag{1}$$

It is natural that the foreshortening is less for a narrow strip. Altschuler and Oliner also gave an empiric formula for the width dependence.

In this short paper a new dynamic method for the calculations of the foreshortening of half-wave stripline resonators will be presented. The method is based on calculations of cavity-backed slot antennas [2], [3]. Fig. 1 shows a cavity for such an antenna. The usual electric wall at the cavity bottom has been changed to a magnetic wall. For a cavity-backed slot antenna an admittance can be defined at the center of the slot. This admittance consists of two parts: one from the exterior region of the cavity and one from the interior region in the cavity. Here we are only interested in the latter part. If we put an identical cavity on the other side of the slot, the slot admittance will be twice the interior admittance of one cavity. On the other hand, the dual configuration of the double cavity-backed slot is just the stripline configuration of Fig. 2. Consequently, by using Babinet's principle we can achieve the input impedance Z_{strip} over the infinitesimal gap in the middle of the stripline resonator, as indicated in Fig. 2, from the interior admittance Y_{slot} of a single cavity

$$Z_{\text{strip}} = \frac{1}{4} 2 Y_{\text{slot}} \frac{\mu_0}{\epsilon_r \epsilon_0} \tag{2}$$

where ϵ_r is the relative dielectric constant of the medium in the cavity, i.e., of the stripline board. Since the medium in the closed box is homogeneous, the field distribution is independent of ϵ_r and so also of the foreshortening. But the impedances and resonance frequencies are of course dependent on ϵ_r. If the magnetic walls in Fig. 2 are moved away from the strip, their influence on the impedance may be neglected.

To get the interior admittance Y_{slot} we use a method [4]–[6] where the different waveguide modes in the cavity build up a proposed electrical field in the slot. At resonance a very good approximation for this field distribution over the slot is

$$E(x, y, 0) = \hat{x} E_0 \cos\left(\pi \frac{y}{a}\right). \tag{3}$$

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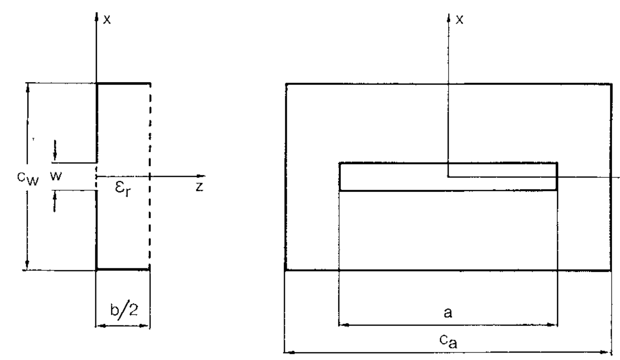

Fig. 1. A modified cavity for a cavity-backed slot antenna. ——— Electric wall. ----- Magnetic wall.

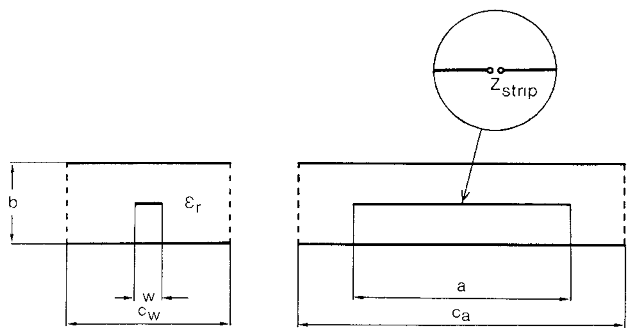

Fig. 2. The half-wave stripline resonator. ——— Electric wall. ----- Magnetic wall.

The transversed field variation is here neglected but will be accounted for later on. The electric-field component along the slot is also neglected. This is an adequate approximation for narrow slots in relation to their lengths.

After lengthy but straightforward manipulations, including a Fourier series expansion of the slot field, the interior admittance is obtained. Since the cavity is assumed to be lossless, the admittance is a pure susceptance

$$B_{\text{slot}} = \frac{32\pi^2 a^2}{k_0 c_a c_w w^2}\sqrt{\frac{\epsilon_0}{\mu_0}}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\epsilon_n \frac{k^2 - k_{my}^2}{k_{mnz}} \cdot \frac{\cos^2\left(k_{my}\frac{a}{2}\right)}{[\pi^2 - k_{my}^2 a^2]^2} \cdot \frac{\sin^2\left(k_{nx}\frac{w}{2}\right)}{k_{nx}^2} \cdot \tanh\left(k_{mnz}\frac{b}{2}\right) \tag{4}$$

where $\epsilon_n = 1$ when $n = 0$ and $\epsilon_n = 2$ when $n \neq 0$, and the propagating constants are

$$k_{mnz} = \sqrt{k_{my}^2 + k_{nx}^2 - k^2} \tag{5}$$

$$k_{my} = (2m+1)\frac{\pi}{c_a} \tag{6}$$

$$k_{nx} = 2n\frac{\pi}{c_w} \tag{7}$$

$$k = \frac{2\pi}{\lambda}\sqrt{\epsilon_r} = k_0\sqrt{\epsilon_r}. \tag{8}$$

The general n-dependent term is

$$T_{mn} = \epsilon_n \frac{\sin^2\left(k_{nx}\frac{w}{2}\right)}{k_{mnz}k_{nx}^2}\tanh\left(k_{mnz}\frac{b}{2}\right). \tag{9}$$

By increasing n this term will approach

$$T_{mn}' = 2\frac{\sin^2\left(n\pi\frac{w}{c_w}\right)}{\left(2\frac{\pi}{c_w}\right)^3 n^3}. \tag{10}$$

The original sum over n will now be replaced by

$$\sum_{n=0}^{\infty} T_{mn} = T_{m0} + \sum_{n=1}^{\infty}(T_{mn} - T_{mn}') + \sum_{n=1}^{\infty} T_{mn}'. \tag{11}$$

The first sum to the right of the equality sign converges rapidly and the second is given by [7]

$$S_1 = \sum_{n=1}^{\infty} T_{mn}' = 2\left(\frac{c_w}{2\pi}\right)^3 \sum_{n=1}^{\infty}\frac{\sin^2(n\pi\delta)}{n^3} = \frac{c_w{}^3\delta^2}{4\pi}\left[\ln\left(\frac{1}{2\pi\delta}\right) + \frac{3}{2} + \frac{(\pi\delta)^2}{36} + \frac{(\pi\delta)^4}{2700} + \frac{(\pi\delta)^6}{79380} + \frac{(\pi\delta)^8}{1701000} + \frac{(\pi\delta)^{10}}{30873150} + \cdots\right] \tag{12}$$

where

$$\delta = \frac{w}{c_w}. \tag{13}$$

These formulas have been obtained on the assumption that the E-field over the slot is constant in the x direction. In the stripline case the E-field over the slot corresponds to the H-field over the strip or, except only for a constant, the current distribution in the strip. Thus a constant E-field is adequate for a shallow cavity or a low-ohmic stripline, since the current distribution in the center conductor of a stripline is

$$I(x) = I(0)\frac{\sinh\left[\frac{\pi}{2}\frac{w}{b}\right]}{\sqrt{\cosh^2\left(\frac{\pi}{2}\frac{w}{b}\right) - \cosh^2\left(\pi\frac{x}{b}\right)}}. \tag{14}$$

For a deep cavity or a high-ohmic stripline (12) should read [8]

$$S_2 = \sum_{n=1}^{\infty} T_{mn}' = \frac{c_w{}^3\delta^2}{4\pi}\ln\frac{1}{\sin\left(\frac{\pi}{2}\frac{w}{c_w}\right)}. \tag{15}$$

This has also been verified by comparing the characteristic impedance of a stripline obtained with the theory of this short paper with the characteristic impedance obtained with conformal mapping. It has then been found that the following weighting between the two sums gives the right characteristic impedance within 0.5 percent

$$S = \sum_{n=1}^{\infty} T_{mn}' = \frac{0.588\left(\frac{w}{b}\right)^2 S_1}{1 + 0.588\left(\frac{w}{b}\right)^2} + \frac{S_2}{1 + 0.588\left(\frac{w}{b}\right)^2}. \tag{16}$$

The strip reactance may now be achieved from (2)

$$X_{\text{strip}} = \frac{1}{2}\frac{\mu_0}{\epsilon_r\epsilon_0}B_{\text{slot}}. \tag{17}$$

A computer program has been carried out in which X_{strip} and its derivative with respect to the frequency ω are determined. The program calculates the resonance frequency, i.e., when $X_{\text{strip}} = 0$. At this frequency a theoretical TEM line without end effects should have a length of half a guide wavelength. The foreshortening is then obtained as the difference between this length and the actual length a.

The impedance behavior of the foreshortened stripline near resonance is similar to that of a theoretical TEM line with no end effects having a characteristic impedance

$$Z_0 = \frac{dX_{\text{strip}}}{d\omega}\frac{\omega}{\pi}. \tag{18}$$

The computer runnings show that this "apparent characteristic impedance" of the foreshortened stripline is less than that obtained from conformal mapping. That means that to get a proposed response of a stripline system, the stubs and resonators should not only be shortened but also narrowed. If the slot has the same length as the cavity, there will be no end effects, and we should get the right characteristic impedance of the stripline. Similarly, if the slot has the same width as the cavity, we can simulate a strip of infinite width and compute its foreshortening. But this has to be taken carefully, since our assumption in (3) is good only for narrow strips.

In the diagram in Fig. 3 the foreshortening is plotted versus the wavelength in the stripline for some widths of the strips. The foreshortening Δa, the stripline wavelength λ_g, and the width w are all normalized with respect to the ground-plane spacing b. The magnetic

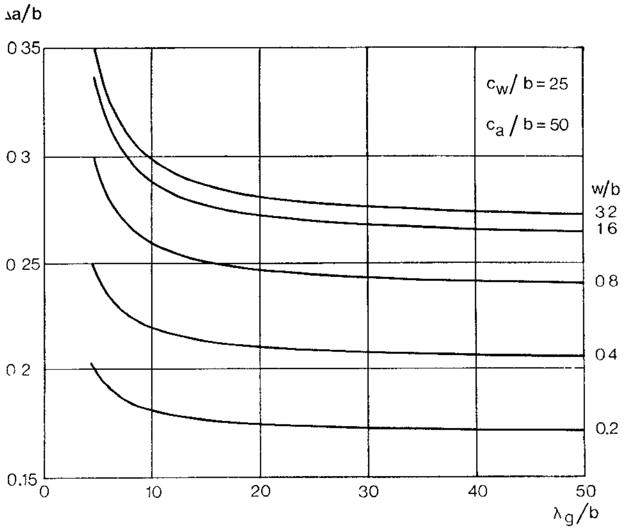

Fig. 3. The foreshortening of a half-wave stripline resonator as a function of the stripline wavelength for some values of the strip width. All dimensions are normalized to the ground-plane spacing.

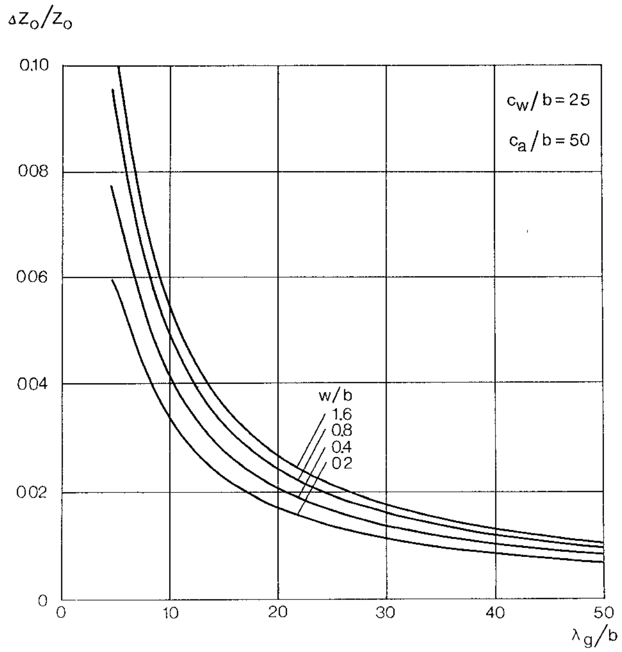

Fig. 4. The relative decrease in the characteristic impedance caused by the end effects of half-wave stripline resonators as a function of the stripline wavelength for some values of the strip width. All dimensions are normalized to the ground-plane spacing.

walls have been moved away far enough so their influence on the results is restricted to the fourth or fifth digit. It is seen that the dynamic foreshortening increases for decreasing resonator length in contrary to the static theory, but is less than the static foreshortening given by (1).

In the diagram in Fig. 4 the relative decrease in the characteristic impedance caused by the end effects is plotted versus the stripline wavelength for some widths of the strips.

The results may also be used for other open-ended stripline configurations, such as $\lambda/4$ stubs, if one takes half the foreshortening in Fig. 3 at every open end of the stripline.

Measurements of the foreshortenings are presented in Fig. 5 together with the corresponding theoretical curves. The foreshortenings have been obtained by measuring the resonance frequencies of stripline resonators. Even if the foreshortening is independent of the relative dielectric constant of the board, the resonance frequency is not. Thus, when we obtain the foreshortenings from the resonance frequencies, the relative dielectric constant is critical. The manufacturer of the stripline board used states that $\epsilon_r = 2.62 \pm 0.05$. The

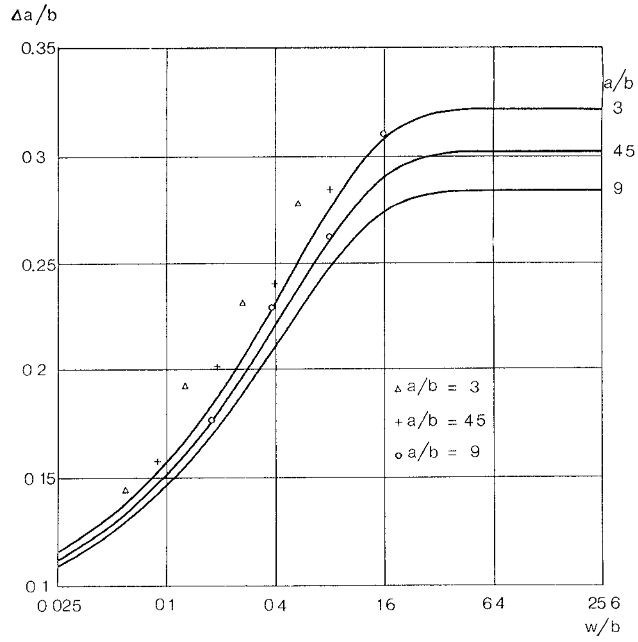

Fig. 5. Measured data points and theoretical curves of the foreshortening as a function of the strip width (logarithmic scale) for some values of the strip length. All dimensions are normalized to the ground-plane spacing.

points have been calculated by using the value 2.62. For narrow strips the agreement is good between theory and practice, but for broader strips the difference is greater. This may depend on the assumption that the current on the strip is laminar all the way to its ends. This is true for a narrow strip, but for a wider strip the current bends to the center at the ends of the strip. This bending makes the current path longer, which will contribute to the foreshortening. Measurements on a slotted strip showed less foreshortening, confirming the current bending theory. Contributory reasons for the difference between theory and practice are the uncertainty of ϵ_r (2.65 had been more advantageous), the finite thickness of the strip, and the air spacing between the boards caused thereby.

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Maximum Phase-Locking Bandwidth Obtainable by Injection Locking

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***Abstract*—A simple rule is presented for the determination of the locking region of an oscillator with a general tuning circuit.**

During the last few years a number of articles have treated the theoretical aspects of injection locking [1]–[6]. Reference is often made to an early paper by Adler [7], whereas the basic work by Van der Pol [8] is often neglected. Van der Pol made a thorough study

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